

Thermodynamics of string black hole with hyperscaling violation

J. Sadeghi ^{*}B. Pourhassan [†] and A. Asadi [‡]

*Sciences Faculty, Department of Physics, Mazandaran University,
P.O.Box 47416-95447, Babolsar, Iran*

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Abstract

In this paper, we start with black brane and construct specific space-time which violates hyperscaling. In order to obtain the string solution we apply Null-Melvin-Twist and KK -reduction. By using the difference action method we study thermodynamics of system to obtain Hawking-Page phase transition. In order to have hyperscaling violation we need to consider $\theta = \frac{d}{2}$. In that case the free energy F is always negative and our solution is thermal radiation without a black hole. Therefore we find that there is not any Hawking-Page transition. Also, we discuss the stability of system and all thermodynamical quantities.

Keywords: String Theory; Black Hole; Hyperscaling Violation; Null-Melvin-Twist; KK-Reduction; Thermodynamics.

1 Introduction

As we know the AdS /CFT correspondence provides an analytic approach to study strongly coupled field theory [1, 2, 3, 4]. Recently, we see several paper about development of AdS gravity theories and their conformal field theory dual, in that case the metric background generalized and it is dual to scale-invariant field theories and not conformal invariant. The scale invariant provided by dynamical critical exponent $z \neq 1$ (the $z = 1$ corresponds to case of the AdS metric) on the following metric, [7],

$$ds^2 = -\frac{1}{r^{2z}}dt^2 + \frac{1}{r^2}(dr^2 + dx_i^2), \quad (1)$$

^{*}Email: pouriya@ipm.ir

[†]Email: b.pourhassan@umz.ac.ir

[‡]Email: ali.asadi89@stu.umz.ac.ir

The corresponding metric will be invariant under following scale transformation,

$$t \rightarrow \lambda^z t, \quad x_i \rightarrow \lambda x_i, \quad r \rightarrow \lambda r. \quad (2)$$

The resulting metric can be a solution of field equations with coupled theories to matter with negative cosmological constant also include an abelian field in the bulk. Space-time metrics that transform covariantly under dilatation have recently been reinterpreted as holography dual to stress tensor of quantum field theories which violates hyperscaling [5, 6]. Recently, the large class of scaling metrics containing an abelian gauge field and scalar dilaton considered [7-19], which is presented by the following equation [11],

$$ds^2 = r^{-2(d-\theta)/d} (r^{-2(z-1)} dt^2 + dr^2 + dx_i^2), \quad (3)$$

where θ is hyperscaling violation exponent. Note that, this metric is not invariant under scale transformation (2), but transforms covariantly as,

$$ds = \lambda^{\theta/d} ds, \quad (4)$$

which defines property of hyperscaling violation in holography language. The corrections of conformal hyperscaling relation in the conformal point of view in large N_f QCD as a concrete dynamical model is given by Ref. [20]. Such examples show that QCD can be a candidate for usage hyperscaling.

On the other hand the Galilean holography developed in Refs. [21, 22], where the non-relativistic generalizations of the AdS/CFT correspondence extracted. An expansion of Galilean algebra can be obtained by adding dilation operator and a special conformal transformation to the time and space scale identically. A discussion of non-relativistic conformal symmetry generalization which is known as Schrödinger has been explained in Ref. [3]. In this discussion, time and space geometry of d dimensions isometry group has Schrödinger symmetry and established over AdS/CFT correspondence. They suggested that the gravity is the holographic dual of the non-relativistic conformal field theories at strong couplings.

The next development of Galilean holographic is finite temperature generalization [23, 24, 25]. In the AdS/CFT correspondence of finite temperature a planar black brane solutions suggested in the Schrödinger space as the holographic dual of the non-relativistic conformal field theory at finite temperature. The investigation of AdS_5 geometry near horizon of D3-brane in flat space is investigated [23, 24]. Then, the known Null-Melvin-Twist (NMT) [26, 27] applied to this system. Ref. [25] started with solution of asymptotical black hole metrics which lead to the string solution and characterize the specific non-relativistic conformal field theories to which they are dual. An analysis of these black hole space-time thermodynamics shows that they describe the dual conformal field theory at finite temperature and finite density. It has been shown that, after doing NMT by applying KK -reduction over S^5 geometry, the result is extremal black brane also the asymptotic limits is reduced to Schrödinger geometry. The thermodynamic solutions of such black hole discussed by Refs. [23, 24].

The new regularization method has been suggested by the Ref. [28] which is the oldest regularization method [29, 30] with some modification which is subtraction method with an

unusual boundary matching.

This paper is organized as the following. In next section we begin with black brane metric and make the corresponding metric which violates hyperscaling. In that case, we apply NMT and KK -reduction and obtain the string solution of this geometry. In section 3 we use the difference action method and extract the thermodynamics of system in section 4. In section 5 we summarized our results.

2 String Black Brane

Now we consider the non-extremal D3-brane geometry [28] near horizon, which is obtained by the following action [21, 24],

$$ds^2 = \left(\frac{r}{R}\right)^2 (-f dt^2 + dy^2 + dx_i^2) + \left(\frac{R}{r}\right)^2 f^{-1} dr^2 + R^2 d\Omega_5^2,$$

$$\phi = 0, \quad B = 0, \quad f(r) = 1 - \left(\frac{r_H}{r}\right)^4, \quad (5)$$

where R is the AdS scale, $x_i = (x_1, x_2)$ and $r = r_H$ is the location of the horizon, so the metric at $r_H = 0$ reduces to the extremal case. ϕ is dilaton and B is $NS - NS$ two-form. A particularly convenient choice for $d\phi$ is given by Hopf fibration $s^1 \rightarrow s^5 \rightarrow P^2$ with the following metric,

$$d\Omega_5^2 = ds_{P^2}^2 + (d\chi + \mathcal{A}), \quad (6)$$

where χ is the local coordinate on Hopf fibre and \mathcal{A} is the one-form on P^2 , and $ds_{P^2}^2$ is metric on P^2 [25]. We need to consider two isometry directions as dy and $d\phi$ for Melvinization process, where dy is along the world-volume, $d\phi$ along the S^5 and y is one of three spatial coordinate. So, the corresponding metric (5) with hyperscaling violation in black hole solution become [7],

$$ds_{d+2}^2 = \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{2\theta/d} \left(-\left(\frac{R^2}{r}\right)^{-2(z-1)} f dt^2 + dy^2 + dx_i^2 + R^2 d\Omega_5^2 \right) + \left(\frac{R}{r}\right)^2 \left(\frac{r_F}{r}\right)^{2\theta/d} f^{-1} dr^2,$$

$$f(r) = 1 - \left(\frac{r_H}{r}\right)^{d+z-\theta}, \quad (7)$$

where $d = 3$ and r_F is scale which is obtained from dimensional analysis [7]. Finite temperature effects in theories with hyperscaling violation studied, in that case in the gravity side we have $r_F < r_h$. From null energy condition (NEC) as $T_{\mu\nu}n^\mu n^\nu \geq 0$ [7, 31] and null vectors satisfy $n^\mu n^\nu = 0$. The above conditions lead us to obtain the following relations,

$$(d - \theta)(d(z - 1) - \theta) \geq 0,$$

$$(z - 1)(d + z - \theta) \geq 0. \quad (8)$$

In order to satisfy our following results with equation (4) we need to consider $z = 1$. From the first relation of (8), one can obtain,

$$(\theta \leq 0, \quad d \geq \theta), \quad or \quad (\theta \geq 0, \quad d \leq \theta). \quad (9)$$

Now, we apply NMT to metric (7) with $z = 1$, we have,

$$\begin{aligned} ds_{d+2}^2 &= K^{-1} \left(\frac{r}{R} \right)^2 M [- (1 + b^2 r^2 M^2) f dt^2 - 2b^2 r^2 f M^2 dt dy + (1 - b^2 r^2 f M^2) dy^2 + K dx_i^2], \\ &+ M \left(\frac{R}{r} \right)^2 f^{-1} dr^2 + MK^{-1} R^2 \eta^2 + MR^2 ds_{p^2}^2, \end{aligned} \quad (10)$$

and

$$\begin{aligned} \phi &= -\frac{1}{2} \ln K, \\ B &= K^{-1} \left(\frac{r}{R} \right)^2 b(f dt + dy) \wedge \eta, \\ K &= 1 - (f - 1)b r^2 M^2, \end{aligned} \quad (11)$$

where $\eta = (d\chi + \mathcal{A})$, $M = \left(\frac{r_F}{r}\right)^{(2\theta/d)}$ and also b has $[L^{-1}]$ dimension. If we perform the KK -reduction on S^5 for the non-extremal solution (10), we obtain

$$\begin{aligned} ds_{d+2}^2 &= K^{-2/3} \left(\frac{r}{R} \right)^2 M [- (1 + b^2 r^2 M^2) f dt^2 - 2b^2 r^2 f M^2 dt dy + (1 - b^2 r^2 f M^2) dy^2 + K dx_i^2] \\ &+ K^{1/3} M \left(\frac{R}{r} \right)^2 f^{-1} dr^2, \end{aligned} \quad (12)$$

and

$$\begin{aligned} \phi &= -\frac{1}{2} \ln K, \\ A &= K^{-1} \left(\frac{r}{R} \right)^2 b(f dt + dy), \end{aligned} \quad (13)$$

where A is one-form field in Einstein frame. It is useful to work following light-cone coordinate,

$$x^+ = bR(t + y), \quad \text{and}, \quad x^- = \frac{1}{2bR}(t - y). \quad (14)$$

So, the solution is,

$$\begin{aligned} ds_{d+2}^2 &= K^{-2/3} \left(\frac{r}{R} \right)^2 M \left[- \left(\frac{f - 1}{(2bR)^2} - \left(\frac{r}{R} \right)^2 f M^2 \right) dx^{+2} - (1 + f) dx^+ dx^- \right. \\ &\quad \left. + (bR)^2 (1 - f) dx^{-2} + K dx_i^2 \right] + K^{1/3} M \left(\frac{R}{r} \right)^2 f^{-1} dr^2, \end{aligned} \quad (15)$$

and

$$\begin{aligned} \phi &= -\frac{1}{2} \ln K, \\ A &= K^{-1} \left(\frac{r}{R} \right)^2 b \left[\frac{f + 1}{2bR} dx^+ + bR(1 - f) dx^- \right]. \end{aligned} \quad (16)$$

The equation (15) is the same as the equation (5) in Ref. [28] with additional $M = \left(\frac{r_F}{r}\right)^{2\theta/d}$. By consideration x^+ coordinate as the time, the recent metric under scale transformation $x^+ \rightarrow \lambda^z x^+, x_i \rightarrow \lambda x_i, r \rightarrow \lambda^{-1}r, x_- \rightarrow \lambda^{2-z}x_-$ and $d = 2\theta$ transforms covariantly as equation (4), and it is violates hyperscaling.

The extremal case coming from $f = 1$ and non-extremal case approaches this at asymptotically large r . The last metric on the light-cone coordinates in equation (14) gives extremal case which is independent of the parameter b . So b is unphysical and thus cannot give any physical quantity. One can interpret this result in the zero-temperature limit [25]. The metric background (15) is a solution of effective action. In non-extremal case for the $\theta = 0$ we have following action [28],

$$S_5 = \frac{1}{16\pi G_5} \int dx^5 \sqrt{-g} \left[\mathcal{R} - \frac{4}{3} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{4} R^2 e^{-8\phi/3} F_{\mu\nu} F^{\mu\nu} - 4A_\mu A^\mu - \frac{V}{R^2} \right], \quad (17)$$

where G_5 , g and R are the 5 dimensional Newton constant, the determinant of 5 dimensional metric and the scalar curvature respectively. $F = dA$ is two-form field and the potential V is defined by following expression,

$$V = 4e^{2\phi/3}(e^{2\phi} - 4). \quad (18)$$

By setting $\phi = 0$, this action reduce to extremal action [21]. As we know in case of $\theta \neq 0$ the shape of action (17) will conserve, but in this process the potentials V and corresponding field ϕ will be changed. Because the K will be changed by parameter of θ .

The ADM form of metric is,

$$\begin{aligned} ds_{d+2}^2 &= K^{1/3} \left(\frac{r_F}{r}\right)^{(2\theta/d)} \left(\frac{R}{r}\right)^2 f^{-1} dr^2 \\ &+ K^{-2/3} \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(2\theta/d)} \left[K dx_i^2 - \left(\frac{1}{(bR)^2(1-f)} + \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(4\theta/d)} \right) f dx^{+2} \right] \\ &+ K^{-2/3} \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(2\theta/d)} \left[(bR)^2(1-f) \left(dx^- - \frac{(1+f)}{2(bR)^2(1-f)} dx^+ \right)^2 \right]. \end{aligned} \quad (19)$$

By using the corresponding metric, we obtain the angular velocity of the horizon Ω_H , that can be interpreted as chemical potential associated with the conserved quantities along the x^- direction,

$$\Omega_H = \frac{1}{2(bR)^2}. \quad (20)$$

Note that we have mentioned two kinds of hypersurfaces; the time-like boundary at a large fixed r and the space-like surface at a fixed time x^+ whose time is described by the *ADM* form. In the extremal case there is problem with g_{--} component in calculation of difference action ($g_{--} = 0$).

3 The Difference Action

The metric (15) gives the extremal solution near the boundary (the large r) and interpreted as the finite temperature generalization of the Galilean holography [23, 24, 25]. We want to consider the thermodynamics of this system in the finite temperature. In order to calculate the thermodynamics, we use difference action method [28, 29, 30].

According to the Ref. [28], first we continue analytically x^+ to ix^+ and put the system into a box by cutoff $r = r_B$. The cutoff r_B is larger than the scale R but it is finite. We subtract the action of the extremal solution from the non-extremal one. We note here each action include two terms such as bulk and Gibbons-Hawking surface term. To do such process, we have to match the geometries of metrics in $r = r_B$ wall. As mentioned earlier, the g_{--} component of the extremal case has been degenerated in metric (15), so we cannot match metrics in the wall. In order to remove this problem, we match the boundary metric of the extremal geometry to the non-extremal one only for the x^- constant. So, we rescale appropriately three dimensional slices (x^+, x^i). We obtain scaled extremal metric as a following,

$$\begin{aligned} ds_{d+2}^2 &= \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(2\theta/d)} \left[\left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(4\theta/d)} H_B^2 dx^{+2} - 2iH_B dx^+ dx^- + G_B^2 dx_i^2 \right] \\ &\quad + \left(\frac{R}{r}\right)^2 \left(\frac{r_F}{r}\right)^{(2\theta/d)} dr^2, \\ \phi &= 0, \\ A &= i \left(\frac{r}{R}\right)^2 \frac{H_B}{R} dx^+, \end{aligned} \tag{21}$$

where

$$\begin{aligned} H_B &= \left[K(r_B)^{-2/3} \left(\frac{h(r_B) - 1}{(2bR)^2} + \left(\frac{r_B}{R}\right)^2 \left(\frac{r_F}{r_B}\right)^{(4\theta/d)} h(r_B) \right) \right]^{1/2} \left(\frac{r_B}{R}\right)^{-1} \left(\frac{r_F}{r_B}\right)^{(-2\theta/d)}, \\ G_B &= K(r_B)^{1/6}. \end{aligned} \tag{22}$$

The difference action ($S - S_0$) will be as,

$$S_0 = S_{0bulk} + S_{0GH}, \quad \text{and,} \quad S = S_{bulk} + S_{GH}, \tag{23}$$

where both S_{0bulk} and S_{bulk} are action (17), but the S_{0bulk} evaluate on the extremal solution (21) and the S_{bulk} calculate on the non-extremal solution (15). Also S_{0GH} and S_{GH} are the Gibbons-Hawking surface term,

$$S_{0GH} = -\frac{1}{8\pi G_5} \int dx^4 \sqrt{g_B} (Tr K_0), \tag{24}$$

where g_B is the determinant of the boundary first fundamental form and $(Tr K_0)$ is the trace of the boundary second fundamental form. We calculate the difference action in limit of $r_B \rightarrow \infty$, which is not divergent,

$$\lim_{r_B \rightarrow \infty} (S - S_0) = \frac{V_4}{16\pi G_5} \frac{r_H^4}{R^5} \left(1 - \frac{\theta}{d}\right) \left(\frac{r_F}{r_H}\right)^{3\theta/d}, \tag{25}$$

where V_4 is volume of four dimensions space-time. It shown that this result agree with Ref. [28] without hyperscaling violation.

4 Thermodynamics

Now, we use results of the previous section to study the thermodynamics of system. In that case the Hawking temperature can be obtained from surface gravity as $\beta = \frac{2\pi}{\kappa}$ where κ is surface gravity,

$$\kappa^2 = -\frac{1}{2} (\nabla^a \xi^b) (\nabla_a \xi_b), \quad (26)$$

where ξ is the killing vector field which is obtained by following expression,

$$\xi = \frac{1}{bR} \frac{\partial}{\partial t} = \partial_+ + \Omega_H \partial_-, \quad (27)$$

and corresponding β is obtained by,

$$\beta = \frac{4}{d+1-\theta} \frac{\pi b R^3}{r_H}. \quad (28)$$

The killing generator of the event horizon (27) not only has components along the boundary time translation direction x^+ , but also along light-like direction x^- . From the gravitational point of view it is therefore a system with chemical potential for x^- directions,

$$\mu = \frac{1}{2(bR)^2}. \quad (29)$$

In order to calculate the thermodynamics of system, we use the following free energy [28]

$$\begin{aligned} F &= -(16\pi G_5) V_3^{-1} \lim_{r_B \rightarrow \infty} (S - S_0) \\ &= -\beta \left(\frac{r_H^4}{R^5} \right) \left(1 - \frac{\theta}{d} \right) \left(\frac{r_F}{r_H} \right)^{3\theta/d} \\ &= -\frac{\pi^4 R^3}{4\mu^2 \beta^3} \left(1 - \frac{\theta}{d} \right) \left(\frac{\beta r_F}{\pi R^2} \right)^{3\theta/d} \left(\frac{4}{d+1-\theta} \right)^{4-3\theta/d} (2\mu)^{3\theta/2d}, \end{aligned} \quad (30)$$

where V_3 is the integration over x^{-i} , and equal to $V_4 \beta^{-1}$. So we obtain entropy as,

$$\begin{aligned} S &= \beta \left(\frac{\partial F}{\partial \beta} \right)_\mu - F \\ &= \frac{4\pi b r_H^3}{R^2} \frac{(1 - \frac{\theta}{d})}{(d+1-\theta)} \left(4 - \frac{3\theta}{d} \right) \left(\frac{r_F}{r_H} \right)^{3\theta/d}. \end{aligned} \quad (31)$$

These equations in case of $\theta = 0$ agree with the Ref. [28]. Also we can obtain,

$$\begin{aligned}
E &= \left(\frac{\partial F}{\partial \beta} \right)_\mu - \mu \beta^{-1} \left(\frac{\partial F}{\partial \mu} \right)_\beta \\
&= \frac{r_H^4}{R^5} \left(1 - \frac{\theta}{d} \right) \left(1 - \frac{3\theta}{2d} \right) \left(\frac{r_F}{r_H} \right)^{3\theta/d}, \\
Q &= -\beta^{-1} \left(\frac{\partial F}{\partial \mu} \right)_\beta \\
&= -\frac{4b^2 r_H^4}{R^3} \left(1 - \frac{\theta}{d} \right) \left(1 - \frac{3\theta}{4d} \right) \left(\frac{r_F}{r_H} \right)^{3\theta/d}.
\end{aligned} \tag{32}$$

In equation (30) we have two condition for F such $F > 0$ and $F < 0$. In case of $F < 0$ we have two conditions as $\theta < 0$ or $\theta > d + 1$, and in case of $F > 0$ we have $d < \theta < d + 1$. So, in case of $\theta = d$ and $\theta = d + 1$ we have Hawking-Page phase transition. As mentioned before we take $\theta = d/2$, so we have always negative F . So, our solution is thermal radiation without a black hole and we have not any Hawking-Page phase transition. As we know in order to calculate the stability of system we need to obtain the Hessian of $\beta(E - \mu Q) - S$ with respect to the thermodynamic variables (r_H, b) and evaluate it at the on-shell values of (β, μ) . In case of $\theta = 0$ it recovers the results of [28]. In case of hyperscaling violation with condition of $\theta > \frac{1}{2}$ the results will be positive and the system is thermodynamically stable. Here, also we check the first law as $dE = TdS + \Omega_H dQ$ and satisfy by the above quantities.

5 Conclusion

In this paper, we considered the black brane metric and made corresponding metric which is violate hyperscaling. By using the difference action method we obtained the thermodynamical quantities such as β, Q, S, E , and F . In case of $F > 0$ we archived two conditions as $\theta < 0$ or $\theta > d + 1$. And also for $F > 0$ we arrived at $d < \theta < d + 1$. Two above condition lead to Hawking-Page phase transition ($\theta = d, \theta = d + 1$). But in this paper we have always negative F because our condition was $\theta = \frac{d}{2}$ and we have not such phase transition. Also we discussed the stability of system which agree with the Ref. [28] in $\theta = 0$. We have shown that in case of hyperscaling violation the θ must be $\theta > \frac{1}{2}$ which is covered by our condition. In general we can say that the system has thermodynamical stability.

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